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DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

COMPROMISE VALUES IN COOPERATIVE GAME THEORY

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Abstract

The aim of this paper is to give a survey on several well-known compromise values in cooperative game theory and its applications.
Special attention is paid to the τ-value for TU-games, the Raiffa-Kalai-Smorodinsky solution for bargaining problems, and the compromise value for NTU-games.

1 Introduction

Since the introduction of cooperative games by von Neumann and Morgenstern in 1944, the problem most extensively studied in cooperative game theory is how to divide the total earnings of the grand coalition if all players cooperate.
Many solution concepts have been proposed to handle these problems. Well-known examples are the core, the Shapley value and the nucleolus in games with transferable utility (TU-games), the core and the Shapley NTU-value in non-transferable utility games (NTU-games), and the Nash bargaining solution in cooperative bargaining theory.

The aim of this paper is to give a survey on a special type of solution concepts, called compromise values. A compromise value is a solution concept which assigns to each game a value that is based on two vectors, the so-called upper and lower values. Prominent examples of compromise values are the τ-value for TU-games, the compromise value for NTU-games, and the Raiffa-Kalai-Smorodinsky solution for bargaining problems.

The paper is organized as follows. First, in section 2 we recall some basic definitions and solution concepts in TU-games. Most attention is paid to the τ-value introduced by Tijs (1981). The τ-value plays a central role in section 3, where we discus several properties and axiomatic characterizations of the τ-value.
In section 4 we consider bargaining problems. Particularly, we are interested in the Raiffa-Kalai-Smorodinsky solution introduced by Raiffa (1953) and characterized by Kalai and Smorodinsky in 1975.
Section 5 is devoted to compromise values in NTU-games. We discuss two extensions of the τ-value to NTU-games introduced by Borm et al. (1992), namely the compromise value and the NTU τ-value.
In section 6 we consider compromise values in several applications of cooperative game theory, and compare the outcomes with outcomes of other economic or game theoretic solution concepts. We consider the following applications in economics and operations research: cost allocation theory, airport games, bankruptcy problems, big boss games, exchange markets, weighted graph games and sequencing games.
Finally, we conclude this paper in section 7 with some remarks and open problems.

2 TU-games

In this section we examine compromise values for TU-games. We start with some basic definitions.

A *transferable utility game* or *TU-game* is an ordered pair (N, v) where N is a finite set of *players* and $v : 2^N \to \mathbf{R}$ is a map assigning to each *coalition* $S \in 2^N$ a real number $v(S)$, called the *worth* of S, and where $v(\emptyset) := 0$.
Often a TU-game (N, v) will be identified with the function v. The class of all TU-games with player set N is denoted by G^N, and by G we denote the class of all TU-games.
A TU-game v is called *convex* if for all coalitions $S, T \in 2^N$

$$v(S) + v(T) \leq v(S \cup T) + v(S \cap T).$$

One of the main topics dealt with in cooperative game theory is, given a game v, to divide the amount $v(N)$ between the players if the grand coalition N is formed.
A *payoff vector* is a vector $x \in \mathbf{R}^N$ which is *efficient*, i.e., $\sum_{i \in N} x_i = v(N)$. Here x_i represents the payoff to player $i \in N$. A payoff vector $x \in \mathbf{R}^N$ is called an *imputation* if $x_i \geq v(\{i\})$ for all $i \in N$. The set of all imputations of the game v is denoted by $I(v)$. The *core of* v is the set

$$C(v) := \{x \in I(v) \mid \sum_{i \in S} x_i \geq v(S) \text{ for all } S \in 2^N\}.$$

If $x \in C(v)$, then no coalition $S \neq N$ has an incentive to split off if x is the proposed payoff vector, because the total amount $x(S) := \sum_{i \in S} x_i$ allocated to S is not smaller than the amount $v(S)$ which they can obtain by forming a subcoalition.
The core of a game can be empty, but it is shown by Shapley (1971) that if v is convex, then $C(v) \neq \emptyset$. Games with a non-empty core are called *balanced*. The class of balanced TU-games with player set N is denoted by B^N.

Since the introduction of TU-games in von Neumann and Morgenstern (1944) many solution concepts have been proposed to allocate the amount $v(N)$ in a fair way between the players. Formally, a *solution concept* on a class $A \subset G$ is a map which assigns to each TU-game $(N, v) \in A$ a vector in $\mathbf{R}^N$ or a set of vectors in $\mathbf{R}^N$. The imputation set and the core are examples of (multivalued) solution concepts. Also many one-point solution concepts, which assign to a game v a unique vector, have been proposed. A one-point solution concept is also called a *rule* or a *value*. The most well-known values are the Shapley value introduced by Shapley (1953) and the nucleolus introduced by Schmeidler (1969).

The *Shapley value* $\Phi(v) \in \mathbf{R}^N$ of a game $v \in G^N$ is a weighted average of the marginal contributions of players to coalitions. Formally, the Shapley value of v is defined by

$$\Phi_i(v) := \sum_{S \subset N \setminus \{i\}} \frac{(|S|-1)!(|N|-1-|S|)!}{|N|!}(v(S \cup \{i\}) - v(S)) \text{ for all } i \in N.$$

The nucleolus is defined on the class of games with non-empty imputation set. Let $v \in G^N$ with $I(v) \neq \emptyset$ and let $x \in \mathbf{R}^N$ and $S \in 2^N$. The *excess of S w.r.t. x*, $E^v(S,x)$, is defined as

$$E^v(S,x) := v(S) - x(S).$$

$E^v(S,x)$ measures the complaint of coalition S against x.
Let $\Theta(x)$ be the $2^{|N|}$-tuple whose components are the excesses $E^v(S,x)$, $S \subset N$, arranged in a nonincreasing order, i.e., $\Theta_i(x) \geq \Theta_j(x)$ whenever $1 \leq i < j \leq 2^{|N|}$. $\Theta(x)$ is the *excess vector (complaint vector)* of x. The *nucleolus* of v, $n(v)$, is the set of all imputations $x \in I(v)$ satisfying

$$\Theta(x) \leq_L \Theta(y) \text{ for all } y \in I(v).[1]$$

So the nucleolus has the property that it minimizes the maximal complaint. Schmeidler (1969) proved that the nucleolus of a game always consists of one point.

A third value for TU-games is the τ-value introduced by Tijs (1981) for quasi-balanced games. The τ-value of a game is a compromise between an upper and a lower value for the game. Let $v \in G^N$ be a TU-game. The vector $M(v) \in \mathbf{R}^N$ with coordinates

$$M_i(v) := v(N) - v(N \setminus \{i\})$$

[1] $\leq_L$ denotes the lexicografic order on $\mathbf{R}^{2^N}$.

is called the *upper value of* v. $M_i(v)$ can be regarded as the maximal payoff player i can expect to get: if he claims more, then it is advantageous for the other players to exclude him from the grand coalition. $M_i(v)$ is also called the *utopia payoff* for player i.
Let $i \in N$ and $S \in 2^N$ with $i \in S$. We calculate what remains for player i if S forms and all other players in S obtain their utopia payoff. The *remainder of* $i \in S$, $R^v(S,i)$, is defined by

$$R^v(S,i) := v(S) - \sum_{j \in S \setminus \{i\}} M_j(v).$$

The vector $m(v) \in \mathbf{R}^N$ with coordinates

$$m_i(v) := \max_{S: i \in S} R^v(S,i)$$

is called the *lower value of* v. $m_i(v)$ denotes the *minimal right* of player i: he can guarantee himself this payoff by offering the members of a suitable coalition S, for which the maximum is achieved, their utopia payoff and then $m_i(v)$ remains for himself.
A game $v \in G^N$ is called *quasi-balanced* iff

$$m(v) \leq M(v) \text{ and } \sum_{i \in N} m_i(v) \leq v(N) \leq \sum_{i \in N} M_i(v).$$

The class of all quasi-balanced games with player set N is denoted by QB^N. That $B^N \subset QB^N$ follows from the following theorem proved by Tijs and Lipperts (1982).

Theorem 2.1. Let $v \in B^N$. Then for all $x \in C(v)$, we have

$$m(v) \leq x \leq M(v).$$

For a game $v \in QB^N$ the τ-*value of* v, denoted by $\tau(v)$, is the unique payoff vector on the line segment in $\mathbf{R}^N$ with end points $m(v)$ and $M(v)$. Thus,

$$\tau(v) := m(v) + \alpha(M(v) - m(v)),$$

where α is such that $\sum_{i\in N} \tau_i(v) = v(N)$.

Example 2.2. Let (N, v) be the 3-person game with $N := \{1,2,3\}$ and

$v(\{1\}) = v(\{2\}) = 0,\ v(\{3\}) = v(\{1,2\}) = 100,$

$v(\{1,3\}) = 200,\ v(\{2,3\}) = 300,\ v(N) = 400.$

Then $M(v) = (100, 200, 300)$,
$m_1(v) = \max\{v(\{1\}),\ v(\{1,2\}) - M_2(v),\ v(\{1,3\}) - M_3(v),\ v(N) - M_2(v) - M_3(v)\} = \max\{0, -100, -100, -100\} = 0$, $m_2(v) = 0$, and $m_3(v) = 100$.
It follows that

$\tau(v) = (0,0,100) + \alpha(100, 200, 200),$

where α is such that $\sum_{i\in N} \tau_i(v) = 400$. Hence, $\alpha = \frac{3}{5}$ and $\tau(v) = (60, 120, 220)$.

Note that for this game $\Phi(v) = (66\frac{2}{3}, 116\frac{2}{3}, 216\frac{2}{3})$ and $n(v) = (50, 125, 225)$.
One easily verifies that in this case the τ-value, the Shapley value and the nucleolus all belong to the core.

Theorem 2.1 illustrates that the τ-value of a balanced game is a compromise between an upper and lower bound for the core. Tijs (1981) gives several classes of TU-games for which these bounds are sharp, e.g. the class of convex games. A compromise value based on sharp bounds for the core is the β-value introduced in Bondareva (1988), and Bondareva and Driessen (1990). For convex games the τ-value and the β-value coincide. Another value for TU-games which is based on lower and upper values is discussed by van Heumen (1984), who uses a (less sharp) upper bound for the core proposed by Milnor (1952). Also van den Brink (1989) considers values for games which are based on upper and lower vectors.
A value for transferable cost games that is based on upper and lower bounds for the core, is the so-called alternate cost avoided (ACA) method. This method, proposed in the 1930's by a consultant of the Tennessee Valley Authority (TVA), will be further studied in section 6.

Driessen and Tijs (1983) provided an alternative approach of calculating the τ-value of quasi-balanced games by introducing the gap function.
Let $v \in G^N$. The *gap function of* v, $g^v : 2^N \to \mathbb{R}^N$, is defined by

$$g^v(S) := \sum_{i\in S} M_i(v) - v(S) \text{ for all } S \in 2^N.$$

The gap $g^v(S)$ of coalition S is the difference between the sum of the utopia payoffs of the players in S and the worth of coalition S. The *concession vector* $\lambda(v) \in \mathbf{R}^N$ is defined by

$$\lambda_i(v) := \min_{S:i\in S} g^v(S) \text{ for all } i \in N.$$

The interest of g^v and the vector $\lambda(v)$ follows from the next theorem.
Theorem 2.3. (Driessen and Tijs (1983))

(i) $\lambda(v) = M(v) - m(v)$ for every $v \in G^N$

(ii) $QB^N = \{v \in G^N \mid g^v \geq 0, \sum_{i\in N} \lambda_i(v) \geq g^v(N)\}$

(iii) If $v \in QB^N$ and $g^v(N) = 0$, then $\tau(v) = M(v)$

(iv) If $v \in QB^N$ and $g^v(N) > 0$, then $\tau(v) = M(v) - g^v(N)(\sum_{i\in N} \lambda_i(v))^{-1}\lambda(v)$.

Using gap functions Driessen and Tijs introduced several interesting classes of quasi-balanced games for which the τ-value is easy to compute. Here we only mention the class of semi-convex games and the class of 1-convex games. For further classes the reader is referred to Driessen (1988).
A game $v \in QB^N$ is called *semi-convex* if $g^v(\{i\}) = \min_{S:i\in S} g^v(S)$ for all $i \in N$. Note that a game $v \in QB^N$ is semi-convex if and only if $m_i(v) = v(\{i\})$ for all $i \in N$. Hence, for semi-convex games the τ-value can easily be determined. It is easy to show that convex games are semi-convex.

Further, a game $v \in QB^N$ is called *1-convex* if $g^v(N) = \min_{S\subset N} g^v(S)$.
Theorem 2.4. (Driessen and Tijs (1983)) If $v \in QB^N$ is 1-convex, then the τ-value and the nucleolus of v both coincide with the barycenter of the core.

Note the resemblance with the result of Shapley (1971) who showed that for convex games the Shapley value coincides with the barycenter of the core.

Furthermore, Driessen and Tijs (1992) extended the τ-value to TU-games with coalition structures. A *coalition structure* in a TU-game is defined to be a partition of the player set. In games with coalition structures it is assumed that instead of the formation of the grand coaltion N, the coalitions in the coalition structure will be formed. Hence, in these games payoff vectors should describe possible divisions of the worth of each coalition in the coalition structure between the members of this coalition. Roughly, the idea behind the τ-value for games with coalition structures is simply to compute separately for each coalition in the coalition structure the τ-value in the subgame induced by this coalition.

We conclude this section with the remark that Tijs and Driessen (1986a) provided an extension of the τ-value from the class of quasi-balanced games to the class of games with a non-empty imputation set, which is based on the principle of imposing taxes on the formation of non-trivial subcoalitions in a multiplicative way. For more details we refer to Tijs and Driessen (1986a) and Driessen (1988). The idea behind this extension plays a role in the paper on linear production games where non-balanced control games are allowed (cf. Curiel et al. (1988)).

3 Properties and characterizations of the τ-value

In this section we investigate several properties of the τ-value on the class of quasi-balanced games. We start with some basic properties.

Proposition 3.1. The τ-value satisifes the following properties on QB^N.

(1) *efficiency:* $\sum_{i\in N} \tau_i(v) = v(N)$ for all $v \in QB^N$.

(2) *individual rationality:* $\tau_i(v) \geq v(\{i\})$ for all $v \in QB^N$ and all $i \in N$.

(3) *the dummy player property:* $\tau_i(v) = v(\{i\})$ for all $v \in QB^N$ and all dummy players i in v, i.e., players $i \in N$ such that $v(S \cup \{i\}) = v(S) + v(\{i\})$ for all $S \subset N \setminus \{i\}$.

(4) *symmetry:* $\tau_i(v) = \tau_j(v)$ for all $v \in QB^N$ and all symmetric players i and j in the game v, i.e., players i and j such that $v(S \cup \{i\}) = v(S \cup \{j\})$ for all $S \subset N \setminus \{i, j\}$.

(5) *covariance*: for all v and all w in QB^N with $w = kv + a$ for some $k \in (0, \infty)$ and $a \in \mathbf{R}^N$ we have $f(w) = kf(v) + a$. (Here the game $kv + a$ is defined by $(kv + a)(S) := kv(S) + a(S)$ for all $S \in 2^N$).

Shapley (1953) showed that the Shapley value is the unique value on G^N which satisfies

the properties (1), (3), (4) and, in addition, *additivity*, which means that the Shapley value of the sum of two games with the same player set is the sum of the Shapley values. However, the Shapley value does not satisfy the individual rationality property. Other characterizations of the Shapley value can be found in e.g. Young (1985a), Hart and Mas-Colell (1989).
On the class of games with non-empty imputation set the nucleolus satisfies all properties mentioned above except additivty. Moreover, the nucleolus is *stable*, i.e., the nucleolus of a game belongs to the core, whenever the core is non-empty. The τ-value and the Shapley value do not satisfy stability. Characterizations of the nucleolus are provided by Snijders (1991), and by Potters (1991).

The rest of this section is devoted to characterizations of the τ-value. First, theorem 3.2 considers several additional properties of the τ-value.

Theorem 3.2. The τ-value satisfies the following properties on QB^N.

(6) *dummy out property:* if $v \in QB^N$ and $D \subset N$ is the set of dummy players in v, then $\tau(v_{|N\setminus D}) = \tau(v)_{|N\setminus D}$. [2]

(7) *complementary monotonicity:* if $v, w \in QB^N$ are such that $v(T) < w(T)$ for some $T \in 2^N$, $T \neq N$, and $v(S) = w(S)$ for all $S \in 2^N$, $S \neq T$, then $\tau_i(v) \geq \tau_i(w)$ for all $i \in N \setminus T$.

(8) *restricted proportionality:* $\tau(v)$ is proportional to $M(v)$ if $m_i(v) = 0$ for all $i \in N$.

(9) *minimal right property:* $\tau(v) = m(v) + \tau(v - m(v))$ for all $v \in QB^N$.

The dummy out property and the complementary monotonicity property for the τ-value are proved in Tijs and Driessen (1986b) and Driessen (1985). Complementary monotonicity of the τ-value means that if a game v is changed to a game w by increasing only the worth of one coalition $T \neq N$ then, according to the τ-value, no player outside T does profit from this deviation. The reader can easily verify that also the Shapley value satisfies the complementary monotonicity property. However, the nucleolus fails to have this property. For a detailed survey of monotonicity properties of the Shapley value, the nucleolus, and the τ-value the reader is referred to Driessen (1985), Otten (1990), and Sagonti (1991).
The restricted proportionality property and the minimal right property are proved in

[2] $v_{|N\setminus D}$ denotes the restriction of v to $N \setminus D$.

Tijs (1987) to provide the following characterization of the τ-value.

Theorem 3.3. (Tijs (1987)) The τ-value is the unique value on QB^N which satisfies efficiency, restricted proportionality and the minimal right property.

Recently, another characterization of the τ-value on QB^N was provided by Calvo et al. (1993). In this characterization three additional properties of the τ-value play a role. Namely, bounded aspirations, convexity, and restricted linearity. It turns out that together with efficiency and covariance these three properties characterize the τ-value on QB^N. For more details on this characterization the reader is referred to Calvo et al. (1993).

The characterizations of Tijs (1987) and Calvo et al. (1993) are characterizations of the τ-value on a fixed player set N. Recently, Driessen provided a characterization of the τ-value on a set of games with a variable number of players, using an axiom of consistency. For more details on this characterization the reader is referred to Driessen (1993).

4 Bargaining problems

Also in bargaining theory a well-known compromise solution appears, i.e., the Raiffa-Kalai-Smorodinsky solution, or shortly, RKS-solution (Raiffa (1953), Kalai and Smorodinsky (1975)). This solution concept plays a central role in this section.
We start with some basic definitions.
A *bargaining problem* for N is a pair (C, d) where $\emptyset \neq C \subset \mathbf{R}^N$, and $d \in \mathbf{R}^N$ are such that

(i) C is closed, convex and *comprehensive*, i.e., if $x \in C$ and $y \in \mathbf{R}^N$ are such that $y \leq x$, then $y \in C$

(ii) $C_d := \{x \in C \mid x \geq d\}$ is bounded

(iii) there is an $x^0 \in C$ with $x^0 > d$.

By BP^N we denote the class of all bargaining problems for N.
The interpretation of a bargaining problem (C, d) is as follows. The players in N try to

reach an agreement on some outcome $x \in C$, yielding utility x_i for player $i \in N$. If the players in N do not reach an agreement, then the *disagreement outcome* d results with utility d_i for player $i \in N$. Condition (iii) implies that the players will have an incentive to reach an agreement. The problem of interest is on which outcome should the players in N agree? Many solutions to handle this problem have been proposed.

A *bargaining solution on* BP^N is a map $f : BP^N \to \mathbf{R}^N$ such that $f(C,d) \in C$ for all $(C,d) \in BP^N$. The most well-known bargaining solution is the Nash bargaining solution introduced by Nash (1950). The *Nash (bargaining) solution* of a bargaining problem $(C,d) \in BP^N$, denoted $N(C,d)$, is the unique point in C_d where the function

$$x \mapsto \prod_{i \in N} (x_i - d_i)$$

is maximal.

An alternative bargaining solution, first proposed by Raiffa (1953), and axiomatically characterized by Kalai and Smorodinsky (1975), is the RKS-solution. This solution is a feasible compromise between the disagreement point and a utopia point.

Let $(C,d) \in BP^N$ be a bargaining problem and let $i \in N$. The *utopia point* for player i is the point

$$u_i(C,d) := \max\{x_i \mid x \in C_d\}.$$

The point $u(C,d) := (u_i(C,d))_{i \in N}$ is called the *utopia point of* (C,d). The *RKS-solution* of (C,d), denoted by $RKS(C,d)$, is defined as the unique weak Pareto optimal point of C lying on the line through d and $u(C,d)$. Here, a point $x \in C$ is called *weak Pareto optimal in (C,d)* if there does not exist a point $y \in C$ with $y > x$. The set of all weak Pareto optimal points in (C,d) is denoted by $WPar(C,d)$.

Example 4.1. Let $N := \{1,2\}$. Consider the bargaining problem (C,d) on N given by $d := (0,0)$ and
$C := \{x = (x_1,x_2) \in \mathbf{R}^N \mid x_2 \leq 4,\ 2x_1 + x_2 \leq 8\}$. See figure 1.

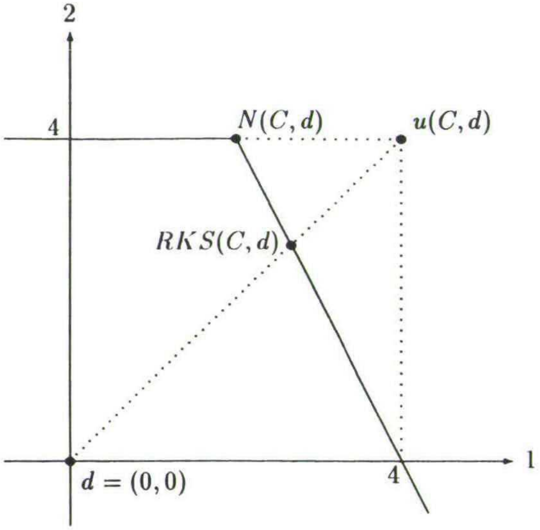

Figure 1.

From figure 1 it immediately follows that $N(C,d) = (2,4)$ and $u(C,d) = (4,4)$. Hence, $RKS(C,d) = (8/3, 8/3)$.

Now we introduce some interesting properties for bargaining solutions.

(i) A bargaining solution $f : BP^N \to \mathbf{R}^N$ is called *Pareto optimal* if for all $(C,d) \in BP^N$ we have $f(C,d) \in Par(C,d) := \{x \in C \mid y \in C, y \geq x \text{ implies } y = x\}$.

(ii) A bargaining solution $f : BP^N \to \mathbf{R}^N$ is called *weak Pareto optimal* if for all $(C,d) \in BP^N$ we have $f(C,d) \in WPar(C,d)$.

(iii) A bargaining solution $f : BP^N \to \mathbf{R}^N$ is called *symmetric* if for all $(C,d) \in BP^N$ with $d_i = d_j$ for all $i,j \in N$ and C such that $(c_i)_{i \in N} \in C$ implies $(c_{\pi(i)})_{i \in N} \in C$ for each permutation π of N, we have $f_i(C,d) = f_j(C,d)$ for all $i,j \in N$.

(iv) A bargaining solution $f : BP^N \to \mathbf{R}^N$ has the *covariance with affine transformations property* if for all $(C,d) \in BP^N$ and all affine functions $A : \mathbf{R}^N \to \mathbf{R}^N$ with $A(x) = \alpha * x + \beta$, $x \in \mathbf{R}^N$, for some $\alpha \in \mathbf{R}^N_{++}$ and $\beta \in \mathbf{R}^N$, we have $f(A(C), A(d)) = A(f(C,d))$. (Here $\alpha * x := (\alpha_i x_i)_{i \in N}$.)

(v) A bargaining solution $f : BP^N \to \mathbf{R}^N$ satisfies *independence of irrelevant alternatives* if for all (C,d), $(D,d) \in BP^N$ with $C \subset D$ and $f(D,d) \in C$ we have $f(C,d) = f(D,d)$.

(vi) A bargaining solution $f : BP^N \to \mathbb{R}^N$ has the *(restricted) monotonicity property* if for all (C, d), $(D, d) \in BP^N$ with $C \subset D$ and $u(C, d) = u(D, d)$ we have $f(C, d) \leq f(D, d)$.

Nash (1950) proved that, in case $|N| = 2$, the Nash solution is the unique bargaining solution which satisfies the properties (i) (or (ii)), (iii)-(v). Later, this result was extended to bargaining problems with more than two players.
The main axiom in this characterization is the axiom of independence of irrelevant alternatives, to which much criticism was raised (see, for example Luce and Raiffa (1957), and Kalai and Smorodinsky (1975)). As an alternative for the independence of irrelevant alternatives axiom, Kalai and Smorodinsky (1975) suggested a monotonicity axiom, which is very much related to the (restricted) monotonicity property (cf. Peters (1992)). The replacement of the independence of irrelevant alternatives axiom by the (restricted) monotonicity property leads to

Theorem 4.2. (cf. Kalai and Smorodinsky (1975)) The RKS-solution is the unique bargaining solution on the class of two player bargaining problems which satisfies the properties (i) (or (ii)), and (iii), (iv) and (vi).

An alternative characterization of the RKS-solution using a reduced game property was obtained by Peters et al. (1991). In this paper also the RKS-solution is implemented by the unique subgame perfect equilibrium of a non-cooperative game in extensive form. Another non-cooperative game leading to the RKS-solution was developed earlier in Moulin (1984).

In the next section we will see that, by weakening some of the properties which characterize the RKS-solution for two player bargaining problems, one can obtain an extension of theorem 4.2 to a class of NTU-games.

5 NTU-games

In this section we consider the more general class of NTU-games introduced by Aumann and Peleg (1960).

A *non-transferable utility game* or *NTU-game* is a pair (N, V), where N is a finite set of players and V is a map assigning to each coalition $S \in 2^N \setminus \{\emptyset\}$ a subset $V(S)$ of $\mathbf{R}^S$ of *attainable payoff vectors.* We assume that for each $i \in N$ there exists a real number $v(i)$ such that $V(\{i\}) = \{x \in \mathbf{R} \mid x \leq v(i)\}$. Further, we assume that for each $S \in 2^N \setminus \{\emptyset\}$ the following properties hold

(i) $V(S)$ is a non-empty, closed and comprehensive subset of $\mathbf{R}^S$

(ii) $V(S) \cap \{x \in \mathbf{R}^S \mid x_i \geq v(i) \text{ for all } i \in S\}$ is bounded.

Similar to TU-games we will identify an NTU-game (N, V) often with V.

The next two examples illustrate that the class of NTU-games comprises the class of TU-games and the class of bargaining problems.

Example 5.1. Let (N, v) be a TU-game. (N, v) gives rise to an NTU-game (N, V), where for each $S \in 2^N \setminus \{\emptyset\}$

$$V(S) := \{x \in \mathbf{R}^S \mid x(S) \leq v(S)\}.$$

Example 5.2. Each bargaining problem (C, d) for N corresponds to an NTU-game (N, V), where

$$\begin{aligned} V(N) &:= C \\ V(S) &:= \{x \in \mathbf{R}^S \mid x \leq (d_i)_{i \in S}\} \text{ for all } S \in 2^N \setminus \{\emptyset, N\}. \end{aligned}$$

In Borm et al. (1992) the compromise value is introduced as an extension of the τ-value to a subclass of NTU-games. Similar to the τ-value for quasi-balanced TU-games the compromise value is based on upper and lower bounds for the core of an NTU-game.

Let (N, V) be an NTU-game. For each $S \in 2^N \setminus \{\emptyset\}$, let

$$dom(S) := \{x \in \mathbf{R}^S \mid x < y \text{ for some } y \in V(S)\}.$$

The elements of $dom(S)$ are elements which are dominated by coalition S.
The *core of* (N, V), denoted $C(V)$, consists of all payoff vectors attainable for the grand coalition N which are not dominated by any coalition S, i.e.,

$$C(V) := \{x \in V(N) \mid (x_i)_{i\in S} \notin dom(S) \text{ for all } S \in 2^N \setminus \{\emptyset\}\}.$$

Let $i \in N$. The *utopia payoff* for player i, $K_i(V)$, is defined by

$$K_i(V) := \sup\{t \in \mathbf{R} \mid \exists_{a\in \mathbf{R}^{N\setminus\{i\}}} : (a,t) \in V(N), a \notin dom(N \setminus \{i\}), a \geq (v(j))_{j\in N\setminus\{i\}}\}.$$

By assumption (ii) in the definition of an NTU-game it follows that $K_i(V) < \infty$. However, it might happen that $K_i(V) = -\infty$. We restrict ourselves to NTU-games (N, V) for which $K_i(V) \in \mathbf{R}$ for all $i \in N$. The vector $K(V) := (K_i(V))_{i\in N}$ is called the *upper value* of V.
Let $i \in N$ and let $S \in 2^N$ with $i \in S$. The *remainder* of $i \in S$ is given by

$$\rho^V(S,i) := \sup\{t \in \mathbf{R} \mid \exists_{a\in \mathbf{R}^{S\setminus\{i\}}} : (a,t) \in V(S), a > (K_j(V))_{j\in S\setminus\{i\}}\}.$$

The *minimal right* of player i is denoted by

$$k_i(V) := \max_{S: i\in S} \rho^V(S,i),$$

and the vector $k(V) := (k_i(V))_{i\in N}$ is called the *lower value* for V. Again, we restrict ourselves to NTU-games (N, V) for which $k(V) \in \mathbf{R}^N$.

Analogously to theorem 2.1 we have
Theorem 5.3. (Borm et al. (1992)) If (N, V) is an NTU-game with $C(V) \neq \emptyset$, then

$$k(V) \leq x \leq K(V) \quad \text{for all } x \in C(V).$$

Moreover, we have
Theorem 5.4. (Borm et al. (1992))

(i) Let (N, v) be a TU-game with $v(N \setminus \{i\}) \geq \sum_{j\in N\setminus\{i\}} v(\{j\})$ for all $i \in N$ and let (N, V) be the corresponding NTU-game. Then $K(V) = M(v)$ and $k(V) = m(v)$.

(ii) Let (C, d) be a bargaining problem for N, and let (N, V) be the corresponding NTU-game. Then $K(V) = u(C, d)$ and $k(V) = d$.

The compromise value is defined on the class of compromise admissible NTU-games. An

NTU-game (N,V) is called *compromise admissible* if

$$k(V) \leq K(V), \quad \text{and } k(V) \in V(N), \; K(V) \notin dom(N).$$

By C^N we denote the class of all compromise admissible NTU-games with player set N. From theorem 5.3 it immediately follows that $V \in C^N$ if $C(V) \neq \emptyset$. Furthermore, from theorem 5.4 it follows that NTU-games corresponding to bargaining situations are compromise admissible, and that for quasi-balanced TU-games (N,v) with $v(N \setminus \{i\}) \geq \sum_{j \in N \setminus \{i\}} v(\{j\})$ for all $i \in N$, the corresponding NTU-games are compromise admissible.

For a compromise admissible NTU-game (N,V) the *compromise value* $T(V)$ is defined as the unique vector on the line segment between $k(V)$ and $K(V)$ which lies in $V(N)$ and is nearest to the utopia value $K(V)$, i.e.,

$$T(V) := k(V) + \alpha_V(K(V) - k(V)),$$

where

$$\alpha_V := \max\{\alpha \in [0,1] \mid k(V) + \alpha(K(V) - k(V)) \in V(N)\}.$$

The following corollary immediately follows from theorem 5.4.

Corollary 5.5. ((Borm et al. (1992))

(i) If v is a quasi-balanced TU-game satisfying $v(N \setminus \{i\}) \geq \sum_{j \in N \setminus \{i\}} v(\{j\})$ for all $i \in N$, and (N,V) is the corresponding NTU-game, then $\tau(v) = T(V)$.

(ii) If (C,d) is a bargaining problem for N, and (N,V) is the corresponding NTU-game, then $RKS(C,d) = T(V)$.

So the compromise value definitionally extends the τ-value and the RKS-solution to NTU-games. As theorem 5.6 and theorem 5.7 below show both the characterization of the τ-value by Tijs (1987) (theorem 3.3) and the characterization of the two player RKS-solution by Kalai and Smorodinsky (1975) (theorem 4.2) can be extended in order to provide characterizations of the compromise value. Therefore we introduce the following properties of values for NTU-games which are straightforward extensions of properties for values for TU-games and solutions for bargaining problems.

Let $f : C^N \to \mathbb{R}^N$ be a value on the set of compromise admissible games with player set N.

(i) f is called *efficient* if $f(V) \in V(N) \setminus dom(N)$ for all $V \in C^N$

(ii) f satisfies the *minimum right property* if $f(V) = k(V) + f(V - k(V))$ for all $V \in C^N$

(iii) f satisfies *restricted proportionality* if $f(V)$ is proportional to $K(V)$ for all $V \in C^N$ with $k(V) = 0$

(iv) f is called *symmetric* if for all $V \in C^N$ and all $i, j \in N$ with $k_i(V) = k_j(V), K_i(V) = K_j(V)$, we have $f_i(V) = f_j(V)$

(v) f is *monotonic* if for all $V, W \in C^N$ with $k(V) = k(W), K(V) = K(W)$ and $V(N) \subset W(N)$ we have $f(V) \leq f(W)$

(vi) f satisfies *covariance* if for all $V \in C^N$, all $\alpha \in \mathbf{R}^N_{++}$ and all $\beta \in \mathbf{R}^N$ we have $f(\alpha * V + \beta) = \alpha * f(V) + \beta$.

Clearly, the compromise value satisfies all properties mentioned above.
It turns out that the first three properties characterize the compromise value on the set $\overline{C}^N \subset C^N$ of all compromise admissible games (N, V) for which the boundary of the set $\{x \in V(N) \mid x \geq k(V)\}$ contains no segments parallel to a coordinate hyperplane.

Theorem 5.6. (Borm et al. (1992)) The compromise value is the unique value on $\overline{C}^N$ which satisfies efficiency, restricted proportionality, and the minimum right property.

The properties (i), (iv)-(vi) characterize the compromise value on the smaller subclass $\tilde{C}^N \subset \overline{C}^N$ of compromise admissible games (N, V) satisfying

(1) $k(V) < K(V)$

(2) $(k_{N\setminus\{i\}}, K_i(V)) \in V(N)$ for all $i \in N$

(3) $V(N)$ is convex.

Theorem 5.7. (Borm et al. (1992)) The compromise value is the unique value on $\tilde{C}^N$ which satisfies efficiency, symmetry, monotonicity and covariance.

Besides the properties mentioned above, the compromise value also satisfies other standard properties, such as individual rationality and the dummy property. Additional properties of the compromise value such as the dummy out property and a complementary monotonicity property which is slightly different from the complementary monotonicity property of the τ-value can be found in Otten (1990). Also an extension of the compromise value to NTU-games with coalition structures can be found in Otten (1990).

Borm et al. (1992) provided another extension of the τ-value to NTU-games, namely the NTU τ-value. The NTU τ-value is based on the same ideas as the Shapley NTU-value (Shapley (1969)). Given an NTU-game, Shapley considered so-called λ-transfer TU-games associated with this NTU-game. The Shapley NTU-value is obtained from the Shapley value of these TU-games. Similarly, the NTU τ-value is obtained from the τ-value of quasi-balanced λ-transfer games.
Let (N, V) be a NTU-game and let $\lambda \in \Delta_N := \{x \in \mathbb{R}^N \mid x \geq 0, \sum_{i \in N} x_i = 1\}$. λ is called *V-feasible* if for all $S \in 2^N \setminus \{\emptyset\}$:

$$v_\lambda(S) := \sup\{\sum_{i \in S} \lambda_i x_i \mid x \in V(S)\} < \infty.$$

So, a V-feasible λ generates a TU-game (N, v_λ). This TU-game is called a *λ-transfer game* corresponding to (N, V). If for all V-feasible $\lambda \in \Delta_N$ the corresponding λ- transfer games are quasi-balanced, the game (N, V) is called *τ-admissible.* For a τ-admissible NTU-game (N, V) the *NTU τ-value*, denoted by $\tau(V)$, is defined by

$$\tau(V) := \{x \in V(N) \mid \text{ there is a } V\text{-feasible } \lambda \in \Delta_N \text{ such that } \tau(v_\lambda) = \lambda * x\}.$$

Note that the NTU τ-value of an NTU-game not necessarily consists of one point, so the name *value* is rather misleading here. The NTU τ-value can even be empty for τ-admissible games. In Borm et al. (1992) a class of τ-admissible NTU-games is given for which the NTU τ-value is nonempty.
If (N, v) is a quasi-balanced TU-game, then the corresponding NTU-game is τ-admissible and the NTU τ-value of the this NTU-game coincides with the τ-value of v. Moreover, for two player bargaining situations the NTU τ-value coincides with the Nash bargaining solution.
An extension of the NTU τ-value to NTU-games with coalition structures can be found in Otten (1990).

6 Applications

Cost allocation problems

In many real life situations the problem of allocating joint costs occurs. Examples are setting fees for common facilities like communication networks, canals, airports etc. Other examples are the allocation of joint costs among the divisions of a firm, and the allocation of costs among the users of a water supply system. A theoretical tool to analyse this type of problems is provided by cooperative game theory.
To formulate a cost allocation problem in terms of cooperative game theory, it is modelled as a *cost game* (N, c), where N represents the set of participants among which the joint costs should be divided, and $c : 2^N \to \mathbf{R}$ is the so-called *(joint) cost function.* For any coalition $S \in 2^N$, $c(S)$ denotes the minimal costs of designing a project only to serve the purposes of the members of S.
Given a cost game (N, c), the cost allocation problem becomes how to allocate the joint costs in a fair way.
For games corresponding to reward situations notions like impututation set, core etc. are important. For games corresponding to cost situations these notions should be reversed. The *reverse-core* of (N, c) is defined by

$$C^r(c) := \{x \in \mathbf{R}^N \mid x(N) = c(N), x(S) \leq c(S) \text{ for all } S \in 2^N\}.$$

The reader easily verifies that $x \in C^r(c)$ if and only if $-x \in C(-c)$.
We say that a cost game (N, c) is *concave* if and only if $(N, -c)$ is convex. Similarly, the notion of τ-value can be adjusted to cost games. We say that a cost game c is *reverse quasi-balanced* if $-c$ is quasi-balanced. The *reverse τ-value*, $\tau^r(c)$, of a cost game (N, c), is defined as $\tau^r(c) := -\tau(-c)$ if $-c$ is quasi-balanced.
Note that for a reversed quasi-balanced cost game (N, c), $\tau^r(c)$ is the unique efficient compromise between the two vectors $M^r(c)$ and $m^r(c)$ defined by

$$\begin{aligned} M_i^r(c) &:= M_i(c) \text{ for all } i \in N \\ m_i^r(c) &:= \min_{S: i \in S} R^c(S, i) \text{ for all } i \in N. \end{aligned}$$

Tijs and Driessen (1986b) introduced the reverse τ-value for cost games using gap functions.

An alternative cost allocation rule related to the reverse τ-value is the so-called *alternate cost avoided method*, or shortly the *ACA-method*. This method, proposed in the 1930's by the Tennessee Valley Authority (TVA) (see Ransmeier (1942), Straffin and Heaney (1981), Young (1985b)), is the unique efficient compromise on the line between the vector $M^r(c)$ and the vector $(c(\{i\}))_{i\in N}$. Hence, the reverse τ-value of a cost game c coincides with the ACA-method if the cost game is such that $m_i^r(c) = c(\{i\})$ for all $i \in N$, i.e., if $-c$ is semi-convex. Aoki (1989) analyses the reverse τ-value for cost games with concave cost functions.
In Otten (1993) two characterizations of the ACA-method are provided, one on a class of cost games with a fixed player set, and one on a class of cost games with a variable player set using a reduced game property.

Airport games

A special type of cost allocation situations is related to airports. Consider the aircraft landing fee problem of an airport with one runway. Suppose that the planes which are to land are classified into m types. Let N_j be the set of landings by planes of type j over a fixed period of time. Then $N := \bigcup_{j=1}^m N_j$ is the set of all landings. Let $n_j := |N_j|$ and $n := \sum_{j=1}^m n_j$.
The cost of building a runway depends on the largest plane for which the runway is designed. Let t_j be the cost to make the runway suitable for landings by planes of type j. We assume that

$$0 =: t_0 < t_1 < t_2 < \ldots < t_m.$$

The cost function $c : 2^N \to \mathbf{R}$ is defined by $c(\emptyset) := 0$ and for $S \in 2^N \setminus \{\emptyset\}$

$$c(S) := \max\{t_j \mid 1 \leq j \leq m, S \cap N_j \neq \emptyset\}.$$

Note that the game c is equal to $\widehat{c}$, where

$$\widehat{c} := \sum_{k=1}^{m}(t_k - t_{k-1})u^*_{\bigcup_{r=k}^m N_r}$$

and for $T \in 2^N$, u^*_T is the game defined by $u^*_T(S) = 1$ if $T \cap S \neq \emptyset$ and $u^*_T(S) = 0$ otherwise.

For the reverse τ-value of the airport game we have (cf. Tijs and Driessen (1986b)) in case $n_m \geq 2$

$$\tau_i^r(c) = t_m(\sum_{k=1}^{m} n_k t_k)^{-1} t_j \quad \text{if } i \in N_j.$$

So the reverse τ-value assigns cost allocations proportional to the cost of a shortest runway needed by a player. The proof of this statement is based on the fact that airport games are concave and so, the reverse τ-value coincides with the ACA-method. Moreover, we have that the marginal cost $M_i^r(c)$ of each player $i \in N$ equals zero. Hence, $\tau^r(c)$ is proportional to

$$(c(\{1\}), \ldots, c(\{n\})) = (t_1, \ldots, t_1, t_2 \ldots, t_2, \ldots, t_m, \ldots, t_m).$$

In Littlechild and Owen (1973) and Dubey (1982) the Shapley value of airport games is discussed and characterized. For the nucleolus of airport games the reader is referred to Littlechild (1974), Littlechild and Owen (1977) and Owen (1982).

Bankruptcy problems

A *bankruptcy problem* is a pair $(E, d) \in \mathbf{R} \times \mathbf{R}^N$, where $d_i \geq 0$ for all $i \in N$ and $0 \leq E \leq \sum_{i \in N} d_i$. Here, E is the estate which has to be divided among the claimants, and d_i is the claim of claimant $i \in N$. Several allocation rules for bankruptcy problems have been proposed. An *allocation rule* is a function f which assigns to every bankruptcy problem (E, d) a vector $f(E, d) \in \mathbf{R}^N$ such that

(i) $0 \leq f_i(E, d) \leq d_i$ for all $i \in N$

(ii) $\sum_{i \in N} f_i(E, d) = E$.

Some examples of allocation rules are the proportional rule, which divides the estate proportional to the claims of the creditors, the constrained equal award rule, and the adjusted proportional rule introduced by Curiel et al. (1987).

The *adjusted proportional rule*, or *AP-rule*, starts by giving each claimant $i \in N$ his *minimal right* m_i, which is the maximum of zero and the amount not claimed by the other claimants, i.e., $m_i := \max\{E - \sum_{j \in N \setminus \{i\}} d_j, 0\}$. Next, the amount of the estate which is left, $E' := E - \sum_{i \in N} m_i$, has to be divided. Because each claimant already received a part of his claim the claims are lowered. The claim of claimant $i \in N$ on E'

becomes $d_i' := \min\{d_i - m_i, E'\}$ (claims higher than E' are considered irrational). Now the remaining estate E' is divided proportionally to the new claims.

Example 6.1. Consider the bankruptcy problem (E, d) with $E = 400$, and $d = (100, 200, 300)$. To determine $AP(E, d)$ we first have to compute the minimal rights of the players.

$m_1 = \max\{400 - 200 - 300, 0\} = 0$,

$m_2 = \max\{400 - 100 - 300, 0\} = 0$, and

$m_3 = \max\{400 - 100 - 200, 0\} = 100$.

The remaining estate $E' = E - \sum_{i \in N} m_i = 300$ and the new claims become $d' = (100, 200, 200)$. Hence,

$$AP(E, d) = (0, 0, 100) + \frac{300}{500}(100, 200, 200) = (60, 120, 220).$$

The AP-rule satisfies several nice properties. Some of them are listed below.

(i) The AP-rule satisfies the *minimal right property*, which states that it makes no difference whether the rule is directly applied to a given bankruptcy situation, or that first the minimal rights are allocated to the players and then the AP-rule is applied on the remaining estate and the adjusted claims.

(ii) The AP-rule is *symmetric*, which means that if two claimants have the same claims, they also receive the same part of the estate.

(iii) The AP-rule satisfies the *truncated claim property*, which means that, given a bankruptcy problem, it does not matter for the allocation if all claims above the estate are replaced by claims equal to the estate.

(iv) The AP-rule satisfies the *additivity of claims property*. This property states that, given a bankruptcy problem (E, d) satisfying $m_i = 0$ for all $i \in N$, if one of the claimants dies leaving behind parts of his claim to different heirs, which become new claimants, this does not affect the allocation to the other claimants.

It turns out that the four properties listed above are sufficient to characterize the AP-rule.

Theorem 6.2. (Curiel et al. (1987)) The AP-rule is the unique allocation rule for bankruptcy problems satisfying the properties (i)-(iv).

For a bankruptcy problem $(E,d) \in \mathbf{R}\times\mathbf{R}^N$, the corresponding *bankruptcy game* $(N, v_{E,d})$ is defined by (cf. O'Neill (1982))

$$v_{E,d}(S) := \max\{E - \sum_{i\in N\setminus S} d_i, 0\} \text{ for all } S \in 2^N.$$

In Curiel et al. (1987) it is shown that bankruptcy games are convex games, and hence, the τ-value can easily be computed.

Example 6.3. Consider the bankruptcy problem (E,d) of example 6.1. The corresponding bankruptcy game $v := v_{E,d}$ is given by

$v(\{1\}) = v(\{2\}) = 0,\ v(\{3\}) = v(\{1,2\}) = 100,$

$v(\{1,3\}) = 200,\ v(\{2,3\}) = 300,\ v(N) = 400.$

In example 2.2 the τ-value of this TU-game is computed. We found that $\tau(v) = (60, 120, 220) \in C(v)$. Hence, the τ-value of this bankruptcy game coincides with the AP-solution of the bankruptcy problem. That this is no coincidence is shown in

Theorem 6.4. (Curiel et al. (1987)) Let (E,d) be a bankruptcy problem and let $(N, v_{E,d})$ be the corresponding bankruptcy game. Then

(i) $AP(E,d) = \tau(v_{E,d})$ and

(ii) $\tau(v_{E,d}) \in C(v_{E,d})$.

An alternative game theoretic approach to bankruptcy problems is introduced by Dagan and Voly (1992).

Given a bankruptcy problem (E,d), one can construct a bargaining problem $(C_{(E,d)}, b_{(E,d)})$ as follows. The most natural choice for the set $C_{(E,d)}$ of feasible outcomes is to define

$$C_{(E,d)} := \{x \in \mathbf{R}^N \mid x \leq d, \sum_{i\in N} x_i \leq E\}.$$

The choice of the disagreement outcome $b_{(E,d)}$ is not as natural as the choice of $C_{(E,d)}$. Dagan and Voly (1992) proposed two possible alternatives: $b_{(E,d)} := 0$, and $b_{(E,d)} := m(E,d)$, where $m(E,d)$ denotes the vector consisting of the minimal rights of the players. In case $b_{(E,d)} = m(E,d)$ we have the following theorem.

Theorem 6.5. (Dagan and Voly (1992)) Let (E, d) be a bankruptcy problem and let $(C_{(E,d)}, m(E, d))$ be the corresponding bargaining problem. Then

$$RKS(C_{(E,d)}, m(E,d)) = AP(E,d) = \tau(v_{E,d}).$$

Exchange markets

Many economic situations can be modelled using cooperative game theory. Sometimes it is more natural to use NTU-games than to use TU-games. This is the case for example if one wants to model exhange markets as cooperative games.

An *exchange market* $\mathcal{E}$ is a tuple $< N,\ \mathbf{R}_+^m,\ (f^i)_{i\in N},\ (u_i)_{i\in N} >$. Here, N is the set of agents, $\mathbf{R}_+^m$ is the commodity space, $f^i \in \mathbf{R}_+^m$ is the initial commodity bundle of agent $i \in N$, and $u_i : \mathbf{R}_+^m \to \mathbf{R}$ is the utility function of agent i. An ***admissible reallocation*** of coalition S is a collection of commodity bundles $(x^i)_{i\in S}$ such that $x^i \in \mathbf{R}_+^m$ for each $i \in S$ and $\sum_{i\in S} x^i = \sum_{i\in S} f^i$. The set of admissible reallocations of coalition S is denoted by $A(S)$.

An exchange market $\mathcal{E}$ gives rise to an NTU-game $(N, V_\mathcal{E})$ as follows. For each $S \in 2^N \setminus \{\emptyset\}$ define

$$V_\mathcal{E}(S) := \{t \in \mathbf{R}^S \mid \exists_{(x^i)_{i\in S}\in A(S)}[u_i(x^i) \geq t_i \text{ for all } i \in S]\}.$$

The following well-known example of Shafer (1980) can be found in Borm et al. (1992).

Example 6.6. Consider the following exchange market $\mathcal{E}$ with three agents and two commodities. The initial commodity bundles of the agents 1,2 and 3, and the utility functions are given by

$$f^1 = (1-\epsilon, 0),\ f^2 = (0, 1-\epsilon),\ f^3 = (\epsilon, \epsilon),$$

$$u_1(x_1, x_2) = u_2(x_1, x_2) = \min\{x_1, x_2\}, \text{ and}$$
$$u_3(x_1, x_2) = \tfrac{1}{2}(x_1 + x_2) \text{ for all } (x_1, x_2) \in \mathbf{R}_+^2$$

where $0 \leq \epsilon < \frac{1}{5}$.

The corresponding NTU-game $(N, V_\mathcal{E})$ is given by

$$V_\mathcal{E}(\{i\}) = \{t \in \mathbf{R} \mid t \leq 0\},\ i = 1, 2$$
$$V_\mathcal{E}(\{3\}) = \{t \in \mathbf{R} \mid t \leq \epsilon\},$$

$$
\begin{aligned}
V_\mathcal{E}(\{1,2\}) &= \{(t_1,t_2) \in \mathbf{R}^2 \mid t_1 + t_2 \le 1-\epsilon,\ t_1 \le 1-\epsilon,\ t_2 \le 1-\epsilon\},\\
V_\mathcal{E}(\{1,3\}) &= \{(t_1,t_3) \in \mathbf{R}^2 \mid t_1 + t_3 \le \tfrac{1}{2}+\tfrac{1}{2}\epsilon,\ t_1 \le \epsilon,\ t_3 \le \tfrac{1}{2}+\tfrac{1}{2}\epsilon\},\\
V_\mathcal{E}(\{2,3\}) &= \{(t_2,t_3) \in \mathbf{R}^2 \mid t_2 + t_3 \le \tfrac{1}{2}+\tfrac{1}{2}\epsilon,\ t_2 \le \epsilon,\ t_3 \le \tfrac{1}{2}+\tfrac{1}{2}\epsilon\},\\
V_\mathcal{E}(\{1,2,3\}) &= \{(t_1,t_2,t_3) \in \mathbf{R}^3 \mid t_1 + t_2 + t_3 \le 1,\ t_1 \le 1,\ t_2 \le 1,\ t_3 \le 1\}.
\end{aligned}
$$

Easy computations yield that in this case the compromise value and the NTU τ-value give the same solution, namely $(\frac{1}{2}-\frac{1}{2}\epsilon, \frac{1}{2}-\frac{1}{2}\epsilon, \epsilon)$.
However, the Shapley NTU-value of this game differs from the compromise value and the NTU τ-value. The Shapley NTU-value gives the outcome $\{(\frac{5}{12}-\frac{5}{12}\epsilon, \frac{5}{12}-\frac{5}{12}\epsilon, \frac{1}{6}+\frac{5}{6}\epsilon)\}$. We see that the Shapley NTU-value always gives a positive payoff to agent 3 of at least $\frac{1}{6}$ even if $\epsilon = 0$. But if $\epsilon = 0$, agents 1 and 2 together can achieve a utility of 1 by forming the subcoalition $\{1,2\}$, leaving 0 for agent 3. This was the reason that Shafer (1980) argued that in this case the Shapley NTU-value is not a reasonable outcome. The compromise value and the NTU-τ-value however, give a utility of 0 to agent 3 if $\epsilon = 0$.

Big boss games

A TU-game (N, v) is called a *big boss game* (with player i as big boss) (cf. Muto et al. (1988)) if and only if the following three conditions hold:

(i) v is *monotonic*, i.e., if $S \subset T \subset N$, then $v(S) \le v(T)$

(ii) $v(S) = 0$ if $i \notin S$

(iii) $v(N) - v(S) \ge \sum_{j \in N \setminus S} M_j(v)$ if $i \in S$.

Condition (i) implies that $v \ge 0$ and that $M(v) \ge 0$. Condition (ii) states that player i is very powerful, i.e., coalitions not containing i cannot get anything, and (iii) implies that for a coalition without the big boss the marginal contribution to the grand coaltion is at least as large as the sum of the marginal contributions of each of its members. It turns out that there are many economic situations which give rise to big boss games. We mention

(1) bankruptcy problems with one big claimant, i.e., a claimant who claims more than the estate

(2) one-seller many buyers situations of a certain type

(3) information market games as introduced in Muto et al. (1989).

For more applications the reader is referred to Muto et al. (1988). Generalizations of big boss games were studied in Potters et al. (1989) (clan games) and Nagarajan (1992) (games with leading coalitions).

In the next theorem some results for big boss games are collected.

Theorem 6.7. Let (N, v) be a big boss game with player i as the big boss. Then

(i) the core of v is a paralellotope, consisting of the vectors $x \in \mathbf{R}^N$ with $\sum_{i \in N} x_i = v(N)$ and $0 \leq x_j \leq M_j(v)$ for all $j \in N \setminus \{i\}$

(ii) the τ-value and the nucleolus of v both coincide with the center of the core, i.e.,

$$\tau_j(v) = n_j(v) = \begin{cases} v(N) - \frac{1}{2}\sum_{k \in N \setminus \{j\}} M_k(v) & \text{if } j = i \\ \frac{1}{2} M_j(v) & \text{if } j \neq i \end{cases}$$

(iii) for the Shapley value $\Phi(v)$ we have $\Phi_i(v) \leq \tau_i(v)$

(iv) $\Phi(v) = \tau(v) = n(v)$ if and only if v is convex.

Weighted graph games

Brown and Housman (1988) introduced weighted graph games as a class of games where the value of a coalition with more than two players depends on the values of the two player coalitions. Formally, a *weighted graph game* is a TU-game (N, v) where

$$v := \sum_{T:|T|=2} \alpha_T u_T$$

with $\alpha_T \geq 0$ for all $T \in 2^N$, $|T| = 2$. Here, u_T denotes the T-*unanimity game* defined by

$$u_T(S) := \begin{cases} 1 & \text{if } T \subset S \\ 0 & \text{otherwise.} \end{cases}$$

A weighted graph game corresponds to a weighted complete graph in which the players are the vertices and the weight on an edge $T \subset N$, with $|T| = 2$ is given by α_T. For a coalition $S \in 2^N$, $v(S)$ can be seen as the sum of the weights on the edges of the subgraph induced by S.

The folowing theorem illustrates that for weighted graph games the Shapley value, the nucleolus and the τ-value coincide.

Theorem 6.8. (Brown and Housman (1988)) Let (N, v) be a weighted graph game. Then for all $i \in N$

$$\Phi_i(v) = \tau_i(v) = n_i(v) = \frac{1}{2}(\text{the sum of the weights of all edges adjacent to } i).$$

As a corollary of this theorem it follows that the τ-value and the nucleolus are additive on the cone

$$K_2^N := cone\{u_T \mid T \in 2^N, |T| = 2\}.$$

In van den Nouweland et al. (1993) it is shown that the τ-value is additive on every cone K_l^N with $2 \leq l \leq |N|$. Here, $K_l^N := cone\{u_T \mid T \in 2^N, |T| = l\}$.

Theorem 6.9. (van den Nouweland et al. (1993)) Let $(N, v) \in K_l^N$ $(2 \leq l \leq |N|)$. Then $\Phi(v) = \tau(v)$.

It is not difficult to show that the nucleolus does not coincide with the τ-value and the Shapley value on K_l^N if $l > 2$.

Sequencing games

In a *sequencing situation* there is a queue, consisting of n customers waiting to be served at a counter. The original order of the customers is given by a permutation π of $N := \{1, \ldots, n\}$. In the sequel we assume w.l.o.g. that $\pi(i) = i$ for al $i \in N$. For every $i \in N$, s_i denotes the *service time* of i and c_i is the *cost function* of i. We assume that c_i is affine, i.e., $c_i(t) = \alpha_i t + \beta_i$ for all $t \in \mathbb{R}_+$.

Given a sequencing situation one can construct a TU-game in the following way (cf. Curiel et al. (1989)). The set of players is N and, we define v in such a way that the worth of a coalition S is equal to the maximal cost savings the coalition can obtain by rearranging their positions in the queue. Hereby, we allow two customers in the queue to change positions only if there is no customer outside S standing between them. The cost savings that two neighbours i and j in the queue can obtain by switching are $g_{ij} := \max\{\alpha_j s_i - \alpha_i s_j, 0\}$.

A coalition $T \in 2^N$ is called *connected* if for all $i, j \in T$ and all $k \in N$, with $i < k < j$, we have $k \in T$. For a connected coalition T the maximal cost savings are

$$v(T) := \sum_{i,j \in T: i<j} g_{ij}.$$

For a non-connected coalition S, we say that $T \subset S$ is a *component* of S if T is connected and $T \cup \{i\}$ is not connected for every $i \in S \setminus T$. The components of S form a partition of S which we denote by $\mathcal{P}(S)$. Now we define

$$v(S) := \sum_{T \in \mathcal{P}(S)} v(T).$$

The game (N, v) defined above is called the *sequencing game* corresponding to a sequencing situation. In Curiel et al. (1989) it is shown that sequencing games are convex games, and therefore, the τ-value can easily be computed. For player $i \in N$ the utopia payoff $M_i(v)$ is equal to

$$M_i(v) = \sum_{j,k \in N:\, j<k} g_{jk} - \sum_{j,k \in N:\, j<k<i} g_{jk} - \sum_{j,k \in N:\, i<j<k} g_{jk} = \sum_{j,k \in N:\, j \neq k,\, j \leq i \leq k} g_{jk}.$$

Since sequencing games are zero-normalized, i.e., $v(\{i\}) = 0$ for all $i \in N$, it follows that the τ-value is proportional to the upper value.

7 Final remarks and open problems

We conclude this paper with some remarks and open problems related to compromise values.

In this paper we studied solution concepts which are based on upper and lower values for games. In section 2 we have seen that the τ-value of a quasi-balanced TU-game can also be seen as a value based on an upper value for the game and a concession vector. Here, the upper value is used as a starting point, which gives more than the worth of the grand coalition to the players, and the concession vector indicates in which way to lower the payoffs in order to reach an efficient outcome. This approach to the τ-value provides a relation with a broad literature on another type of solution concepts for cooperative games, called concession methods. Characteristic for concession methods are an upper or lower value as a starting point and a concession vector which indicates how the payoffs in the starting point should be lowered or increased in order to reach an efficient outcome. Well-known examples of concession methods are the egalitarian solution for bargaining problems, and the egalitarian nonseparable cost method for cost allocation problems. For literature on this type of solution concepts we refer to Kalai (1977), Peters (1992), Driessen (1988), Driessen and Funaki (1993a, 1993b).

The τ-value gave rise to the introduction of several interesting subclasses of TU-games such as semi-convex games and 1-convex games. Can the compromise value also generate in some way interesting classes of NTU-games ? What is the NTU-analogue of the gap function ?

As mentioned in section 4 the RKS-solution is implemented by subgame perfect equilibria of non-cooperative games in extensive form by Moulin (1984) and Peters et al. (1991). It is still an open problem whether it is possible to implement the τ-value, the compromise value and the NTU τ-value by means of a non-cooperative game in extensive form.

An extension of the τ-value to games with a continuum of players is not known (cf. Aumann and Shapley (1974)).

Characterizations of the Shapley NTU-value are provided by Aumann (1985) and Kern (1985). It is still an open problem whether replacing of one or more of the axioms in these characterizations by suitable axioms for the NTU τ-value will yield a characterization of the NTU τ-value.

A reduced game property for the RKS-solution is provided by Peters et al. (1991), and a reduced game property for the τ-value is given by Driessen (1993). Reduced game properties for the compromise value and the NTU τ-value are still unknown.

In this paper we studied several classes of games for which the τ-value coincided with the Shapley value or the nucleolus. In particular, for weighted graph games al three solution concepts coincide. Brown and Housman (1988) also provided weaker conditions for coincidence of the three solution concepts. It is an interesting problem to find necessary and sufficient conditions for coincidence of the τ-value , the nucleolus and the Shapley value.

In section 6 we have seen that for sequencing games the τ-value is easy to compute. However, for other combinatorial games such as flow games (Kalai and Zemel (1982), Curiel et al. (1989)), traveling salesman games (Fishburn and Pollak (1983), Tamir (1989), Potters et al. (1992)), and minimum spanning tree games (Granot and Huberman (1981)) no explicit formulas for the τ-value are known. For a recent survey on combinatorial optimization games the reader is referred to Tijs (1992).

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